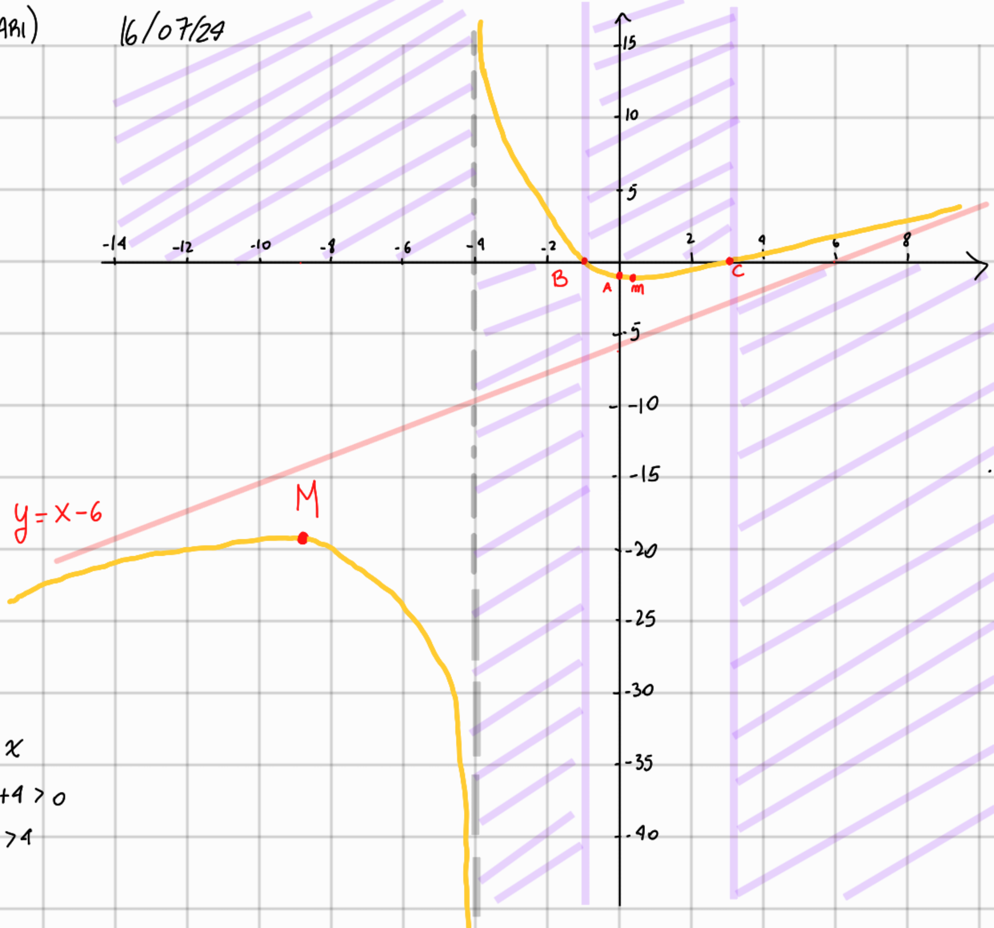
ESAME MATEMATICA E STATISTICA (DISPARI) 16/07/24

ESERCIZIO 1



ⓐ $$f(x) = \frac{x^2-2x-3}{x+4}$$

■ DOMINIO : $x+4 \neq 0 \rightarrow x \neq -4$

$$D = (-\infty, -4) \cup (4, +\infty)$$

■ INTERSEZIONE ASSE y

$$\begin{cases} x = 0 \\ y = \frac{0^2-2\cdot0-3}{0+4} = -\frac{3}{4} \end{cases} \quad A = \left(0, -\frac{3}{4}\right)$$

■ STUDIO DEL SEGNO E INTERSEZIONE ASSE x

$$\frac{x^2-2x-3}{x-4} \geq 0$$

• $x^2-2x-3=0$

$\Delta = 4+12 = 16$

$x_{1,2} = \frac{+2 \pm 4}{2}$ → $x_1 = -1$, $x_2 = 3$

$x \leq -1 \lor x \geq 3$

• $x+4 > 0$

$x > 4$

$$\begin{array}{ccccccc} - & -4 & + & -1 & - & 3 & + \end{array}$$

f è POSITIVA per $x \in (-4,-1) \cup (3,+\infty)$

f è NEGATIVA per $x \in (-\infty,-4) \cup (-1,3)$

f è NULLA per $x = -1 \lor x = 3$

$B = (-1, 0)$ $\quad C = (3, 0)$

■ LIMITI e ASINTOTI

• $$\lim_{x \to -4^-} \frac{x^2-2x-3}{x+4} = \frac{16+8-3}{0^-} = \frac{21}{0^-} = -\infty$$

• $$\lim_{x \to -4^+} \frac{x^2+2x-3}{x+4} = \frac{21}{0^+} = +\infty$$

$\Rightarrow \boxed{x=-4}$ ASINTOTO VERTICALE COMPLETO

• $$\lim_{x \to -\infty} \frac{x^2-2x-3}{x+4} = -\infty$$

• $$\lim_{x \to +\infty} \frac{x^2-2x-3}{x+4} = +\infty$$

(per ordine infiniti) $\Rightarrow$ NO ASINTOTO ORIZZONTALE

CONTROLLO ASINTOTO OBLIQUO

• $$\lim_{x \to \infty} \frac{x^2-2x-3}{x+4} \cdot \frac{1}{x} = \lim_{x \to \infty} \frac{x^2-2x-3}{x^2+4} = 1 = m$$

• $$\lim_{x \to \infty} \frac{x^2-2x-3}{x+4} - x = \lim_{x \to \infty} \frac{\cancel{x^2}-2x-3-\cancel{x^2}-4x}{x+4} = \lim_{x \to \infty} \frac{-6x-3}{x+4} = -6 = q$$

$\boxed{y = x-6}$ ASINTOTO OBLIQUO

DERIVATA PRIMA

$$f'(x)=\frac{(2x-2)(x+4)-(x^2-2x-3)\cdot 1}{(x+4)^2}=\frac{2x^2-\cancel{2x}+8x-8-x^2+\cancel{2x}+3}{(x+4)^2}=$$

$$=\frac{x^2+8x-5}{(x+4)^2}\geq 0$$

21

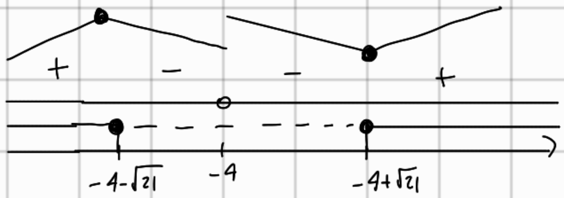
- $x^2+8x-5\geq 0$

$\Delta=64+20=84 \qquad \sqrt{84}=\sqrt{4\cdot 21}=\sqrt{4}\cdot\sqrt{21}=2\sqrt{21}$

$$x_{12}=\frac{-8\pm 2\sqrt{21}}{2} \;\begin{cases} x_1=-4-\sqrt{21}\simeq -8.6 \\ x_2=-4+\sqrt{21}\simeq +0.6\end{cases}$$

- $(x+4)^2>0$

$x\neq 4$



$f(x)$ è CRESCENTE $x\in(-\infty,-4-\sqrt{21})\cup(-4+\sqrt{21},+\infty)$

$f(x)$ è DECRESCENTE $x\in(-4-\sqrt{21},-4)\cup(-4,-4+\sqrt{21})$

$M=(-4-\sqrt{21},\,-2\sqrt{21}-10)$ p.to di massimo

$$f(-4-\sqrt{21})=\frac{(-4-\sqrt{21})^2-2(-4-\sqrt{21})-3}{-\cancel{4}-\sqrt{21}+\cancel{4}}=$$

$$=\frac{16+21+8\sqrt{21}+8+2\sqrt{21}-3}{-\sqrt{21}}=\frac{42+10\sqrt{21}}{-\sqrt{21}}\cdot\frac{(-\sqrt{21})}{(-\sqrt{21})}=-\frac{\overset{2}{42}\sqrt{21}+\overset{10}{210}}{21}=-2\sqrt{21}-10\simeq -19.2$$

$m\,(-4+\sqrt{21},\,2\sqrt{21}-10)$ p.to di minimo

$$f(-4+\sqrt{21})=\frac{16+21-8\sqrt{21}+8-2\sqrt{21}-3}{\sqrt{21}}=\frac{42-10\sqrt{21}}{\sqrt{21}}=2\sqrt{21}-10\simeq -0.8$$

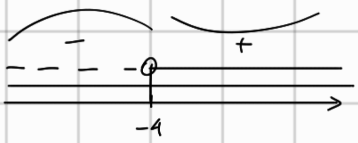
DERIVATA SECONDA

$$f''(x)=\frac{(2x+8)(x+4)^{\cancel{2}}-2(x^2+8x-5)\cancel{(x+4)}}{(x+4)^{\cancel{4}\,3}}=\frac{\cancel{2x^2}+\cancel{8x}+\cancel{8x}+32-\cancel{2x^2}-\cancel{16x}+10}{(x+4)^3}=\frac{42}{(x+4)^3}$$

- $42\geq 0 \quad \mathbb{R}$
- $(x+4)^3>0$

$x+4>0$

$x>-4$



f è CONCAVA per $x\in(-\infty,-4)$

f è CONVESSA per $x\in(-4,+\infty)$

NON CI SONO PUNTI DI FLESSO

(b) $f''(0)=\dfrac{42}{4^3}=\dfrac{21}{32}\Rightarrow$ La funzione in $x_0=0$ sarà convessa poiché questo valore è positivo

(c) Una funzione $f:D\to\mathbb{R}$ è CONTINUA in $x_0\in D$ se $\lim_{x\to x_0} f(x)=f(x_0)$

ESERCIZIO 2

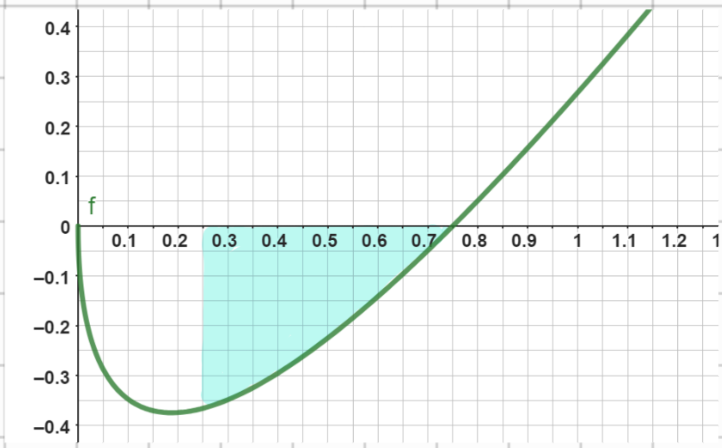
a) $\int \left(2x-\sqrt{3x}\right)dx = \int 2x\,dx - \sqrt{3}\int x^{\frac{1}{2}}dx = x^2 - \sqrt{3}\,\frac{x^{3/2}}{3/2} + c = x^2 - 2\frac{\sqrt{3}}{3}x\sqrt{x} + c$

b) $\int_{1/4}^{3/4} \left(2x-\sqrt{3x}\right)dx = \left[x^2 - \frac{2}{3}x\sqrt{3x}\right]_{1/4}^{3/4} = \frac{9}{16} - \frac{1}{16} - \frac{2}{3}\left[\frac{3}{4}\sqrt{3\frac{3}{4}} - \frac{1}{2}\sqrt{3\frac{1}{4}}\right]$

$= \frac{1}{2} - \frac{2}{3}\left[\frac{9}{8} - \frac{1}{8}\sqrt{3}\right] = \frac{1}{2} - \frac{3}{4} + \frac{1}{12}\sqrt{3}$

$= -\frac{1}{4} + \frac{1}{12}\sqrt{3} \simeq -0.1$

c)



ESERCIZIO 3

$$E(t) = \ln\left(t^2 - 2t + 3\right)$$

Per trovare il p.to di minimo calcolo la derivata prima e ne studio il segno

$$D\left[\ln\left(f(x)\right)\right] = \frac{f'(x)}{f(x)}$$

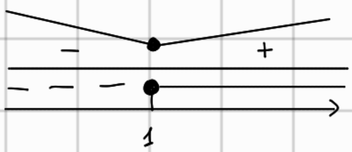
$\Downarrow$

$$E'(t) = \frac{2t-2}{t^2-2t+3} \geq 0$$

- $2t-2 \geq 0$
  $t \geq 1$

$t^2-2t+3 > 0$

$\Delta = 4-12 = -8 < 0 \Rightarrow \forall t \in \mathbb{R}$



$$E(1) = \ln(1-2+3) = \ln(2) \simeq 0.69$$

Il minimo viene raggiunto all'istante $t_{min} = 1$ con $E(t_{min}) \simeq 0.69$

ESERCIZIO 4

X	1960	1970	1980	1990	2000	2010	2020
Y	16.6	16.7	16.1	17.2	17.7	16.9	17.8

- $\bar{y} = \frac{16.6+16.7+16.1+17.2+17.7+16.9+17.8}{7} = \frac{119.0}{7} = 17.0\,°C$

RIORDINO I DATI PER LA MEDIANA

$y^{(1)}$	$y^{(2)}$	$y^{(3)}$	$y^{(4)}$	$y^{(5)}$	$y^{(6)}$	$y^{(7)}$
16.1	16.6	16.7	$\boxed{16.9}$	17.2	17.7	17.8

M = 7 DISPARI

↳ $\text{mediana}(Y) = y^{\left(\frac{M+1}{2}\right)} = y^{(4)} = 16.9$

$$\sigma_y^2 = \frac{1}{7}\left[(-0.4)^2+(-0.3)^2+(-0.9)^2+(0.2)^2+(0.7)^2+(-0.1)^2+(0.8)^2\right]$$
$$= \frac{1}{7}\left[0.16+0.09+0.81+0.04+0.49+0.01+0.64\right] =$$
$$= \frac{2.24}{7} = 0.32$$

- $\alpha = \frac{\sigma_{xy}}{\sigma_x^2}$; $\beta = \bar{y} - \alpha\bar{x}$

$$\bar{x} = \frac{1}{7}\left[1960+1970+1980+1990+2000+2010+2020\right] = 1990$$

$$\sigma_{xy} = \frac{1}{7}\left[(-0.4)(-30)+(-0.3)(-20)+(-0.9)(-10)+(0.2)(0)+(0.7)(10)+(-0.1)(20)+(0.8)(30)\right]$$
$$= \frac{1}{7}\left[12+6+9+7-2+24\right] = \frac{56}{7} = 8$$

$$\sigma_x^2 = \frac{1}{7}\left[(-30)^2+(-20)^2+(-10)^2+0^2+(10)^2+(20)^2+(30)^2\right] = \frac{900+400+100+100+400+900}{7} = \frac{2800}{7} = 400$$

$$\alpha = \frac{\sigma_{xy}}{\sigma_x^2} = \frac{8}{400} = \frac{2}{100} = 0.02$$

$$\beta = 17 - 0.02 \cdot 1990 = 17 - 39.8 = -22.8$$

LA RETTA DI REGRESSIONE LINEARE È

$$\boxed{y(x) = 0.02\,x - 22.8}$$

La temperatura stimata per il 2050 è $y(2050) = 0.02 \cdot 2050 - 22.8 = 41 - 22.8 = 18.2\,°C$